Name: _____

COGS Q350 Mathematical Foundations of Cognitive Science
Practice Exam 3 and Study Hints – Fall 2015

Part 1 of Exam 3 will consist of proofs, both formal and/or informal. (20%)

It is recommended that you study by doing practice proofs, such as those at http://cogs.indiana.edu/q350/deduction1.html and http://cogs.indiana.edu/q350/deduction3.html.

Note that sample answers can be found at http://cogs.indiana.edu/q350/deduction2.html and http://cogs.indiana.edu/q350/deduction4.html .

Focus particularly on proofs involving sub-proofs and proofs involving Disjunctive Syllogism and Modus Tollens, such as most of the proofs at http://cogs.indiana.edu/q350/deduction3.html .

For additional practice, give a formal proof of the following DeMorgan's Law. Prove the law; do not simply cite the law.

Given: A ^ B
Prove: ¬(¬A v ¬ B)

Give an informal proof of the following DeMorgan's Law. Prove the law; do not simply cite the law.

Given: A ^ B
Prove: ¬(¬A v ¬ B)

Part 2 of the exam is a short section requiring you to demonstrate your ability to do matrix multiplication. (5%)

Use matrix multiplication to determine the product.

$$\begin{vmatrix} 1 & 2 & -1 \\ 0 & 3 & 0 \end{vmatrix} * \begin{vmatrix} 2 \\ 1 \\ 3 \end{vmatrix}$$

Part 3 will require you to demonstrate your understanding of neural network learning. (35%)

Consider the procedure below we discussed in class to train a neural network to correctly output the truth values for truth tables. This procedure (quoted) will be described verbatim on the test as follows.

"In order to train neural networks, we will extract a random row of a truth table, let the network calculate the truth value for that row of the truth table. If the network has made a mistake, we will modify the weights. If no error is made, the weights stay the same. Then we randomly choose another row and repeat the process.

x1	x2	Desired Output	Output	outError	What We Will Do	outError * (x1,x2)
1	1	1	-1	1	Raise both weights	(1,1)
1	1	-1	1	-1	Lower both weights	(-1,-1)
1	-1	1	-1	1	Raise w1, lower w2	(1,-1)
1	-1	-1	1	-1	Lower w1, raise w2	(-1,1)
-1	1	1	-1	1	Lower w1, raise w2	(-1,1)
-1	1	-1	1	-1	Raise w1, lower w2	(1,-1)
-1	-1	1	-1	1	Lower both weights	(-1,-1)
-1	-1	-1	1	-1	Raise both weights	(1,1)

Start with randomly generated weights, w1 and w2, in the range [-1, 1].
Start with a threshold value and a learn rate.
outError = (desiredOutput - output) / 2

Repeat the following:

Randomly choose a row in the truth table.
Calculate net = (w1,w2) * (x1,x2)
If net is greater than the threshold value, then output 1, otherwise output -1. That is, output = If[net > threshold, 1, -1].
If output is not = desiredOutput, calculate new weights:
The formula for the new weights is:

(w1,w2) += learnrate*outError*(x1,x2)

where **outError = (desiredOutput - output) / 2**. So

(w1,w2) += learnrate * [(desiredOutput - output) / 2] * (x1,x2)

We continue iterations until the output equals the desired output for all 4 rows of the truth table."

The test question will ask you to teach the neural network to output truth values for disjunction or for conjunction. The weights and threshold value will be given. The specific randomly chosen row in the truth table will be given. You will calculate one iteration, similar to the following example. The information in bold is subject to change on the exam. The rest of the question will be asked as below.

Our task is to teach a neural network to output truth values for **conjunction**.

Let w1 and w2 be randomly chosen weights such that **w1=0.3 and w2=0.8**
Let the threshold value = **0.3**
Let the learnrate = 0.2

Calculate Iteration 1:
We randomly choose a row in the truth table, say **row 3. Then (x1,x2) = (-1, 1).** Use the procedure above to calculate new weights or to show the weights are to remain the same for this iteration. Show every step of the calculation. Explain as needed.

We often speak of neural networks 'learning' when a more accurate term would be neural network 'training.' Explain (in 150 words or less and to someone who knows the basics of neural networks but nothing about how neural networks learn) why the term 'training' is more accurate than 'learning.'

Explain in 150 words or less why the learn rate of 0.2 in the neural network training process above is a reasonable choice. Why is it better than 0.00002? Why is it better than 2? Would 0.1 be reasonable? Why or why not? Be specific and explain your answer in detail.

Part 4 will consist of problems requiring you to demonstrate your ability to use Bayes Rule and basic statistics to solve problems. In addition to the problems we solved in class, practice the following. You may use a calculator. (40%)

Consider the following data set.

A = {-1, 2, -2, 4, -3}

Calculate the standard deviation of data set A. Show all your work. Your answer may be in fraction or decimal form.

The mean of a data set of specimens is 8.5mm, and the standard deviation is 0.5mm. Your lab partner measured a new specimen and told you that its length is less than the mean and is between 2 and 3 standard deviations, inclusive, from the mean. What is the range of values, in mm, that that the new specimen could measure?

Use Bayes' Theorem

$$p(A|B) = \frac{p(A)\,p(B|A)}{p(B)}$$

or a diagram to calculate the following.

Suppose there are two types of stimuli in an experiment: black stimuli and colored stimuli. The black stimuli consists of characters, specifically 15 letters and 20 numbers. The colored stimuli consists of characters, specifically 20 letters and 30 numbers.

Now suppose you choose one of the types of stimuli at random and, select a character at random. The character is a number. What is the probability that it is black? What is the probability that it is colored?

A lab obtains its slides (large and small) from two different suppliers, A and B. The manufacturer gets 50% of its slides from A and 50% of its test tubes from B. 75% of the slides from A are large, and 25% of the slides from B are large. If a particular slide at the lab is large what is the probability that it came from supplier A?